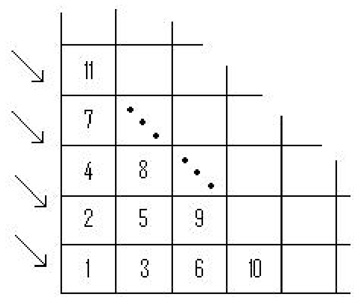
9th Annual Eastern Oregon University

Mathematics Competition Exam – 2012



1. The lower left corner of this grid has coordinates $(1,1)$. Positive integers are placed consecutively following the pattern above. So, for example, the number 9 is in the square with coordinates $(3,2)$. Let i and j be positive integers. Determine, as a polynomial function of i and j, the value in the square with coordinates (i,j).

2. Determine, with proof, which is larger: 100^{100} or 101^{99}?

3. A rearangement of $\{1,2,3,4,5,6\}$ is a six-digit number containing each of those six digits exactly once. (So 265134 is one example of such a rearangement). There are $6! = 720$ such rearangements in all. What is the sum of all 720 of them?

4. Find all solutions to the inequality:

$$\frac{5}{x+1} < \frac{2}{x-1}$$

5. Do there exist positive real numbers a, b, and c for which both equations

$$a^2 + b^2 = c^2 \qquad \text{and} \qquad a^3 + b^3 = c^3$$

are simultaneously true? If so, find such numbers. If not, explain why.

6. Define a polynomial $p(x) = x^3 - 3x^2 - 9x + C$ where C is a real-valued constant. Find all values of C for which the equation $p(x) = 0$ has exactly two real solutions.

7. Prove that

$$sin(x) \geq \int_0^x \sqrt{1 - r^2}\, dr$$

for all x in the range $0 \leq x \leq 1$.