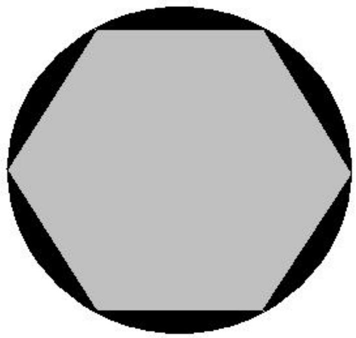
8th Annual Eastern Oregon University Mathematics Competition Exam – 2011



1. What fraction of a circle is taken up by an inscribed regular hexagon?

2. The quantity $\sqrt{23 + 4\sqrt{30}}$ can be re-written without nested square roots. Specifically, the quantity is equal to an expression of the form $\sqrt{a} + \sqrt{b}$ for some particular integer values of a and b. Find the integer values of a and b that make these two expressions equal.

3. Prove that the quantity $(x^{18} + x^{8})$ is always at least as large as the quantity $(x^{16} + x^{10})$ for all real values of x.

4. Suppose Adam's house is one mile due north from Carol's house and suppose Brian's house is one mile due north-east from Carol's house. Suppose we draw a circle of radius one mile around Adam's house. Let P represent the point on the circle that is closest to Brian's house. What is the distance from Brian's hosue to P? Give your answer in simplified form not containing any trig functions.

5. Compute the sum $\sum\limits_{n=1}^{\infty} \frac{1}{\sum_{j=1}^{n} j}$.

(over for additional questions)

6. An "Egyptian fraction" is a rational number which can be expressed as a sum of reciprocals of distinct positive integers. For example, $\frac{4}{5}$ is an Egyptian fraction since it is the sum of the reciprocals of 2, 4, and 20. In other words, $\frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20}$.

The integers must be distinct. For example it wouldn't be legitimate to write $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$. However, $\frac{2}{3}$ is still an Egyptian fraction because $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$.

Prove that if a rational number is of the form $\frac{2}{N}$ where N is an integer larger than 2, then it is an Egyptian fraction.

7. Is it possible for a twice-differentiable function $f(x)$ for which $f(0) = 2$ to satisfy the following condition for all real values of x?

$$f(x)f''(x) - f(x)f'(x) + f'(x)f'(x) = 0$$

If so, find such a function. If not, explain why not.