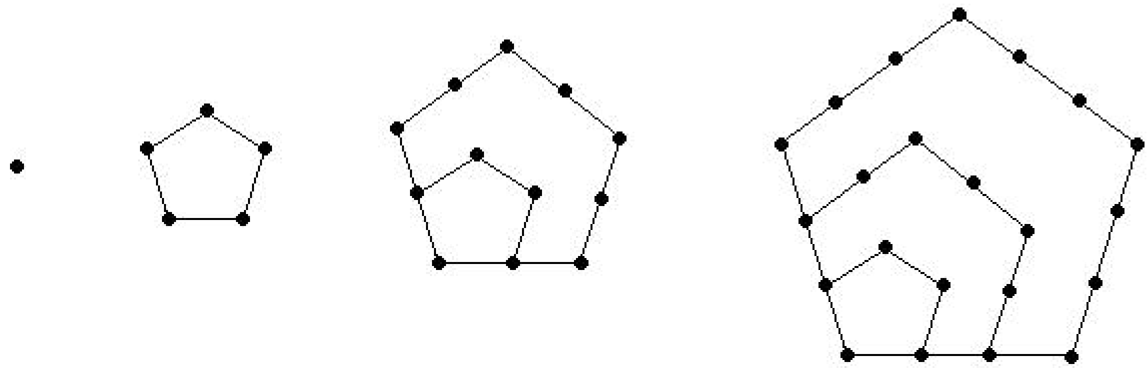

7th Annual Eastern Oregon University Mathematics Competition Exam – 2010

1. The numbers 1, 5, 12, and 22 are called the $1^{st}, 2^{nd}, 3^{rd}$,and 4^{th} "pentagonal numbers" because they are the number of dots in a filled pentagon with side length 1, 2, 3, and 4, respectively (see the picture above). Find, with proof, a formula for the n^{th} pentagonal number (as a function of n).

2. How many positive integers are there which are less than or equal to $35,000$, and which are divisible by 7, but not divisible by 5.

3. Jar one contains a black marble and a red marble. Jar two contains two black marbles and one red marble. Jar three contains three black marbles and two red marbles. First a marble is drawn from the first jar and placed in the second jar. Then a marble is drawn from the second jar and placed in the third jar. Finally, a marble is drawn from the third jar and placed in the first jar. After this process is completed, which is more likely - that jar one will contain two balls of the same color, or that jar one will contain two balls of different color, or are these two possibilities equally likely? (Assume that every time a marble is drawn it is selected at random with any marble in the jar as likely to be selected as any other).

4. Does there exist a positive integer n for which $n^2 + 5n + 8$ is a perfect square? Either find such an n or explain why no such positive integer exists.

(over for more problems)

5. It is a fact that there exist real numbers a and b so that, for any polynomial $P(x)$ of degree 3 or less the following equation is true:

$$\int_{-1}^{1} P(x)dx = P(a) + P(b)$$

Note: The claim is that a and b do not depend on the choice of polynomial $P(x)$.

5a. If this is the case, determine what the values of a and b must be.

5b. Prove the original statement, that the integral above has the given value for any polynomial of degree 3 or less.

6. Define a sequence as follows: $a_1 = 2$; and $a_{n+1} = a_n^2 - a_n + 1$ for $n > 1$. Prove that if $m \neq n$ then a_m and a_n have a no prime factors in common (in other words, prove that their greatest common divisor is 1).

7. Compute the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ exactly.